8. (a) Consider an ideal classical gas, an ideal Fermi gas and an ideal Bose gas. Which of these gases will exert the maximum and the minimum pressure and why ?

(b) Show that the energy density of photons inside a cavity with fixed volume is proportional to T^4. 5+15

Exam. Code : 209003
Subject Code : 4884

M.Sc. Physics 3rd Semester (Batch 2020-22)

STATISTICAL MECHANICS

Paper—PHY-505

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Attempt **FIVE** questions in all, selecting at least **ONE** question from each section. The fifth question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. Two different systems separately in equilibrium have fixed number of particle and energy. When brought in contact with each other so that they can exchange only energy, show that equilibrium is obtained when

$$\beta_1 = \beta_2, \text{ where } \beta = \frac{\partial \ln \Omega}{\partial E}.$$

Where Ω represents number of microstates and other terms have their usual meaning. Further, show that how does this conclusion lead to one of the fundamental relation connecting microscopic and macroscopic world. Comment on your results. 12+5+3

2. In continuation with Q. 1, if we assume that the wall separating the two subsystems is movable as well as conducting but still assumed to be impenetrable to particles, show that equilibrium is attained when

$$\beta_1 = \beta_2, \text{ where } \beta = \frac{\partial \ln \Omega}{\partial E} \text{ and } \eta_1 = \eta_2,$$

where $\eta = \dfrac{\partial \ln \Omega}{\partial V}$. 20

SECTION—B

3. A classical ideal gas with N non-interacting particles is confined to a volume V. Show that the partition function can be written as :

$$Q_N(V, T) = \frac{1}{N!}\left[\frac{V}{\lambda_T^3}\right]^N,$$

where $\lambda_T = \dfrac{h}{\sqrt{2\Pi mkT}}$ is called thermal wavelength, m is mass of each particle and other symbols have their usual meaning. Further obtain an expression for Helmholtz free energy for this system and show that internal energy per atom i.e.,

$$\frac{E}{N} = \frac{3}{2}kT.$$

10+5+5

4. Show that for a classical ideal gas

$$\frac{S}{Nk} = \left(\ln\frac{Q_1}{N}\right) + T\left(\frac{\partial \ln Q_1}{\partial T}\right)_P$$

where S, N, k and Q_1 are entropy, number of particles, Boltzman constant and partition function respectively. 20

SECTION—C

5. For an electron moving in a magnetic field, show that expectation value of $\sigma_z = \tanh \beta\mu_B B$, where, $\beta = 1/k_B T$ and μ_B is Bohr magneton. 20

6. For a free particle in a box, show that

$$\langle H \rangle = -\frac{\partial \ln \mathrm{Tr}\,(e^{-\beta H})}{\partial \beta}.$$

20

SECTION—D

7. Discuss the phenomena of Bose-Einstein condensation. Is it similar to the condensation of vapours into a liquid in ordinary space ? Show that for $T < T_C$ (T_C is the critical temperature) the pressure exerted by a bose condensate is proportional to $T^{5/2}$. 20